Section 17.4 Exercise 5

(a) $x = 29 - 5p + 4q$ price A = p

$y = 16 + 4p - 6q$ price B = q

costs for firm A = $5 + x$

costs for firm B = $3 + 2y$

$\max_{x,y}$ profits $= px + qy - (5+x) - (3+2y)$

will not work

$\downarrow$

express profits in terms of p & q.

$\pi(\underbrace{p, q}_{\text{prices}})$

(b) Firm A $\pi_A(p) = px - (5+x)$

q is taken as given.

$= p(29 - 5p + 4q) - (5 + 29 - 5p + 4q)$

$= 29p - 5p^2 + 4pq - 34 + 5p - 4q$

$= 34p - 5p^2 + 4pq - 34 - 4q$

$\max_p \pi_A(p)$

$\frac{\partial \pi_A}{\partial p} = 34 - 10p + 4q = 0$

$-10p = -34 - 4q$

$p = \frac{17}{5} + \frac{2}{5}q$

$\frac{\partial^2 \pi_A}{\partial p^2} = \ldots$

reaction functions.

$\max_q \pi_B(q)$

$q = \frac{7}{3} + \frac{1}{3}p$

(c) $p = \underbrace{\frac{17}{5}}_{=3\frac{2}{5}} + \frac{2}{5}q$ ✓

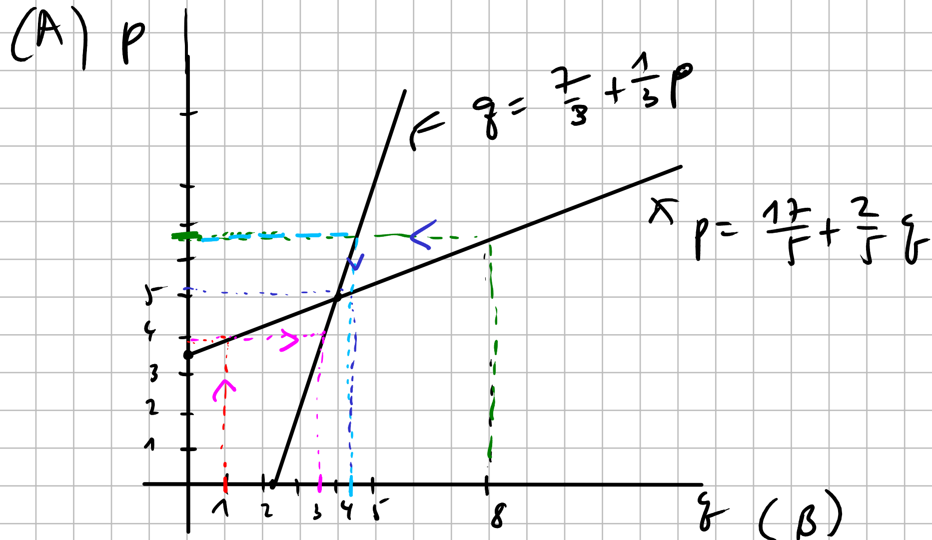
$q = \underbrace{\frac{7}{3}}_{=2\frac{1}{3}} + \frac{1}{3}p$ ✓

$p, q = ?$

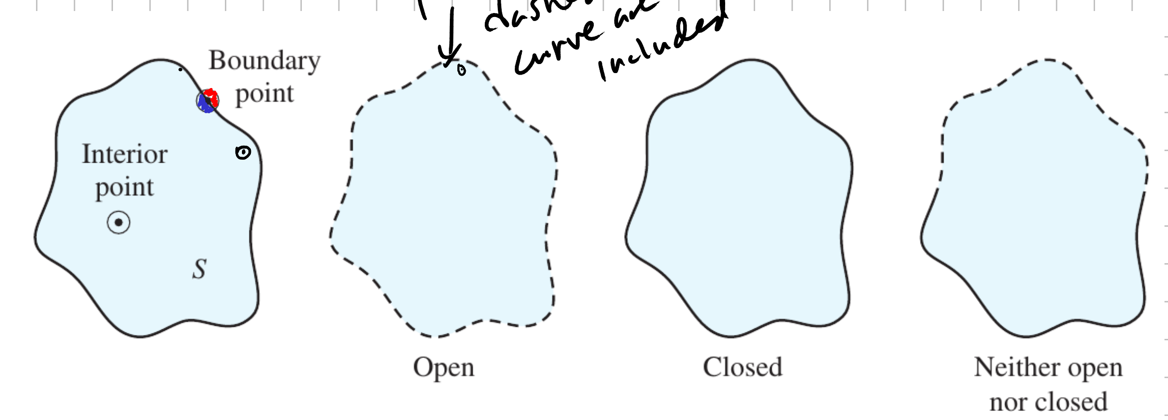
(d)

equilibrium $p = 5$

equilibrium $q = 4$



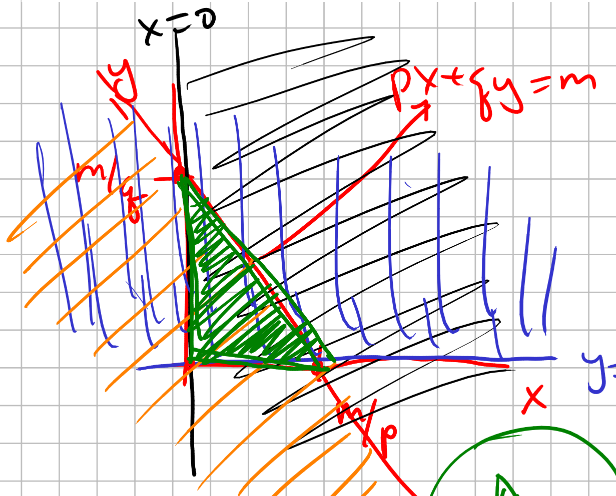
points on dashed curve are not included



Open

Closed

Neither open nor closed

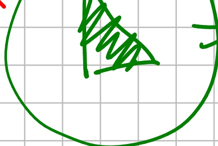


open & closed are not opposites. They are technical terms!

$m \geq 0$

$px+qy \leq m$

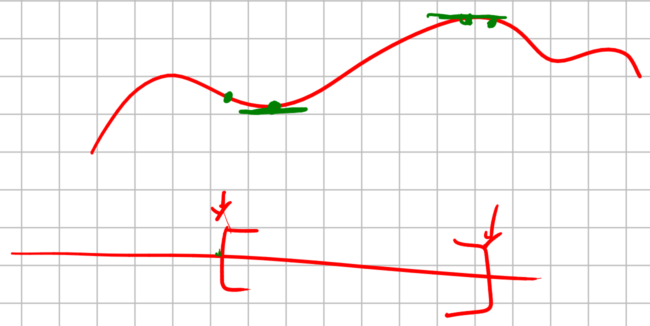
$(0,0) \Rightarrow px+qy = 0 \leq m$



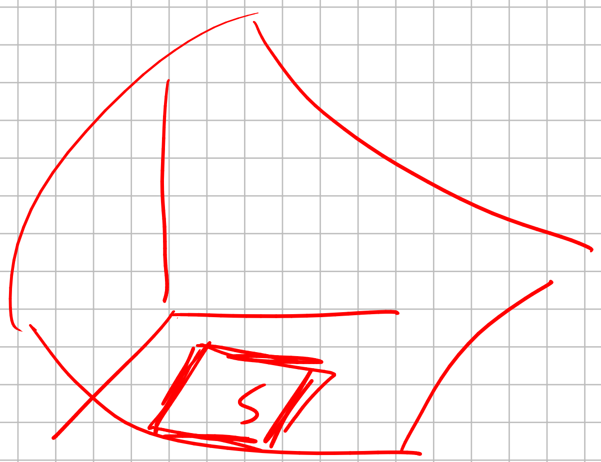
$= \{(x,y): x \geq 0, y \geq 0, px+qy \leq m\}$

$p, q, m > 0$

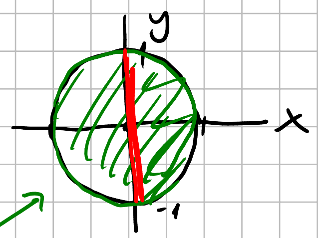
Ch8



Ch17



$x^2+y^2=1$



$x^2+y^2 \le 1$

$(0,0)$

$0^2+0^2 = 0 \le 1$ ✓ true.

$f(x,y) = \boxed{x^2+y^2} + y - 1$

$S = \{(x,y): x^2+y^2 \le 1\}$

$g(y) = 1 + y - 1$

Boundary $x^2+y^2=1$

incorporate

incorporate the fact that we are at the boundary!

$g(y) = y$ $\quad -1 \le y \le 1$ → now at the boundary

Find max/min. Function of one variable along $[-1,1]$

Apply recipe from Ch8 Ch8!

Find stationary pts of g:

$g'(y)=1$ is never zero!

Therefore, there are no interior stationary pts of g.

interior stationary pt of f

$(0,-\frac{1}{2})$

$f(0,-\frac{1}{2}) = -\frac{5}{4}$

Endpoints

$g(-1) = -1$

$g(1) = 1$.

$y=-1 \to$ min

$y=1 \Rightarrow$ max

$y=-1 \Rightarrow$ Because $x^2+y^2=1$, $x=0$ $\quad f(0,-1)=-1$

$y=1 \Rightarrow$ Because $x^2+y^2=1$, $x=0$ $\quad f(0,1)=1$

$\Rightarrow$ $(0,-\frac{1}{2})$ is the minimum pt of f on S.

$(0,1)$ is the maximum pt of f on S.

$x^2+y^2=1 \Rightarrow y^2=1-x^2 \to y=+\sqrt{1-x^2}$, $y=-\sqrt{1-x^2}$

$x^2=1-y^2$ $\quad f(x,y) = x^2+y^2+y+1$

This is not recommend.

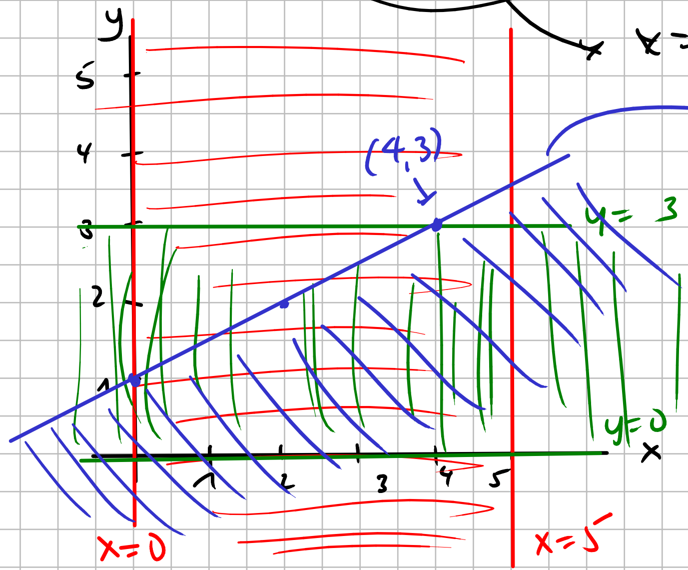
Section 13.5 Exercise 3

$S = \{(x,y) \mid 0\le x\le 5,\ 0\le y\le 3,\ -x+2y\le 2\}$

$\to x=0,\ x=5$

$0\le x\le 5$

$x\ge 0$ & $x\le 5$

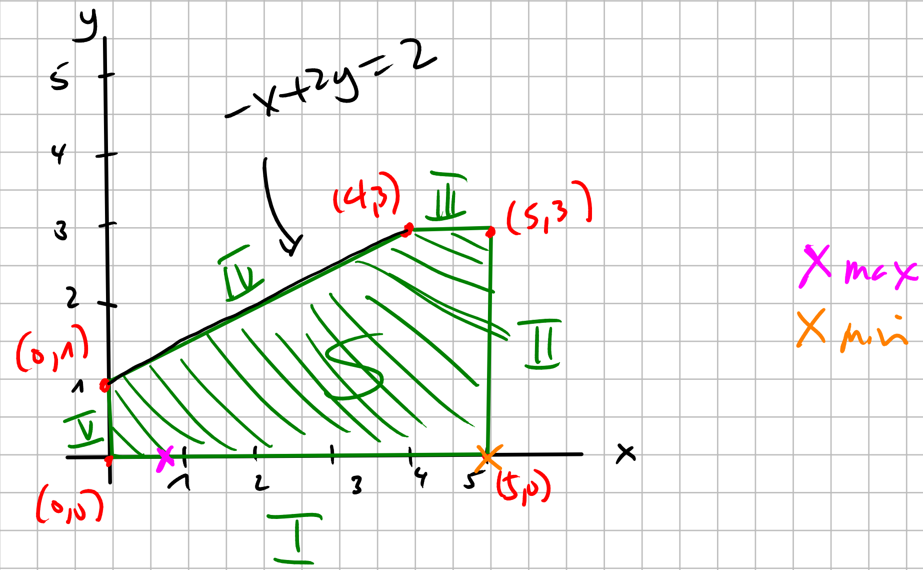


$-x+2y=2$

$(0,1)$ $\quad (2,2)$

$-x+2y\le 2$

$(0,0)$ $\quad -0+2(0)=0\le 2$



$$f(x,y) = 9x + 8y - 6(x+y)^2$$

Step 1 Find stationary pts of f.

$$f'_x = 9 - 12(x+y)(1) \qquad 9 - 12(x+y) = 0$$
$$f'_y = 8 - 12(x+y)(1) \qquad 8 - 12(x+y) = 0$$

$$\Rightarrow x+y = 9/12 \text{ and } x+y = 8/12$$

System of linear equations does not have a solution!

Therefore, f does not have interior stationary points.

Step 2. Find max along every boundary!

Boundary represented by $I = \{(x, 0) : 0 \le x \le 5\}$ y

$$f(x,y) = 9x + 8y - 6(x+y)^2$$

on the boundary $\Rightarrow$ $f(x,0) = 9x - 6x^2$

Function of one variable!

Apply Ch 8 recipe.

- Find stationary points.

$9 - 12x = 0 \Rightarrow x = 3/4$ inside

$f(\frac{3}{4}, 0) = 9(\frac{3}{4}) - 6(\frac{3}{4})^2 = \frac{27}{4} - \frac{27}{8} = \boxed{\frac{27}{8}}$

- Endpoints $x = 0,\ x = 5$

$f(0,0) = 0 \qquad f(5,0) = 45 - 6(5)^2 = \boxed{-105}$

$(\frac{3}{4}, 0)$ is the max pt on boundary represented by I.
$(5, 0)$ is the min pt " " " " " " .

Boundary represented by $\underline{IV}$

$= \{(x,y) : \underline{-x+2y=2}, \ 0 \leq x \leq 4, \ 1 \leq y \leq 3\}$

$f(x,y) = 9x + 8y - 6(x+y)^2$

$\max f(x,y)$
$-x+2y=2$
$0 \leq x \leq 4$
$1 \leq y \leq 3$

Two approaches:
- Change everything in terms of x

$-x+2y=2 \Rightarrow 2y = 2+x$
$y = 1 + \frac{1}{2}x$

$g(x) = 9x + 8(1+\frac{1}{2}x) - 6(x+1+\frac{1}{2}x)^2$
$= 9x + 8 + 4x - 6(\frac{3}{2}x+1)^2$
$= 13x + 8 - 6(\frac{3}{2}x+1)^2$

Apply Ch 8 recipe.

Find stationary pts of g:

$13 - 12(\frac{3}{2}x+1)(\frac{3}{2}) = 0$
$13 - 18(\frac{3}{2}x+1) = 0$
$13 - 27x - 18 = 0$
$-27x = 5$
$x = -5/27$

But $0 \leq x \leq 4$, therefore this is not on the boundary.
Therefore, $x = -5/27$ is not a candidate for max.

Endpoints: $x = 0$ & $x = 4$

$g(0) = \bigcirc =$
$g(4) = \bigcirc =$