Section 13.2 Example 1

$$f(x,y) = 3xy - x^2 - y^2$$

$$f'_x = 3y - 2x$$

$$f'_y = 3x - 2y$$

check this! Here $(0,0)$ is the only stationary point.

$f''_{xx} = -2 < 0$ $f''_{xy} = 3$

$f''_{yy} = -2 < 0$ $f''_{xx} f''_{yy} - (f''_{xy})^2$

$= (-2)(-2) - (3)^2$

$= 4 - 9 = -5 < 0$!

Thm 13.2.1 does not apply.

$$f(x,0) = -x^2$$
$$f(0,y) = -y^2$$

Section 13.4 Example 1

$b_1 > 0$ $b_2 > 0$

$P_1 = a_1 - b_1 Q_1$ $P_2 = a_2 - b_2 Q_2$

$C(Q) = \alpha (Q_1 + Q_2)$

$$Q_1^* = \frac{\boxed{a_1 - \alpha}}{\boxed{2b_1}} \qquad Q_2^* = \frac{\boxed{a_2 - \alpha}}{\boxed{2b_2}}$$

$2b_1 > 0$, $2b_2 > 0$

To guarantee that $Q_1^*, Q_2^* \geq 0$, we must have $a_1 \geq \alpha$ and $a_2 \geq \alpha$.

$$P_1^* = \frac{1}{2}(a_1 + \alpha) \geq \frac{1}{2}(\alpha + \alpha) = \alpha \Rightarrow P_1^* \geq \alpha$$

because $a_1 \geq \alpha$

selling product above per unit cost

Section 13.4 Example 2

(b) Must create an integrated market demand function

$$P_1 = P_2 = P$$

$P = P_1 = 100 - Q_1$

$P = P_2 = 80 - Q_2$

$\Rightarrow P = 100 - Q_1 \Rightarrow Q_1 = 100 - P$

$P = 80 - Q_2 \Rightarrow Q_2 = 80 - P$

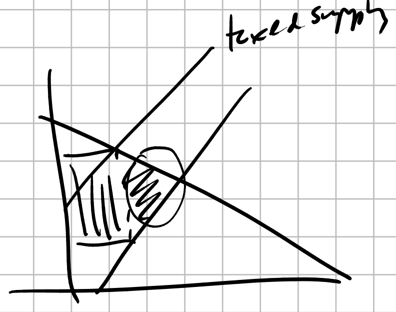
How to combine these two markets $\rightarrow$ What is the total demand?

Q = Total demand $= Q_1 + Q_2 = 100 - P + 80 - P$

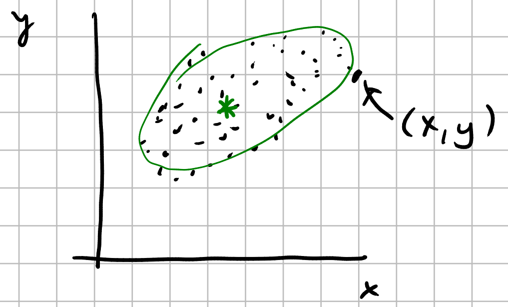
$$Q = 180 - 2P$$

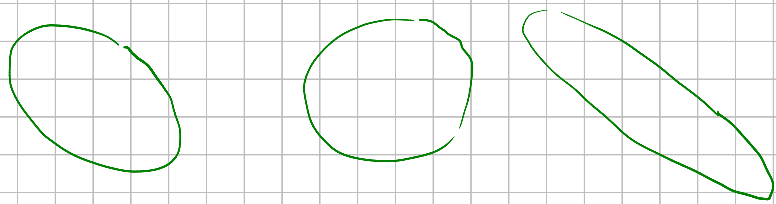
$$Q = 180 - 2P \overset{\text{in terms of } Q}{\Rightarrow} P = 90 - \frac{1}{2}Q$$

$\pi(Q)$



Section 13.4 Example 4





"Summarize" these points.

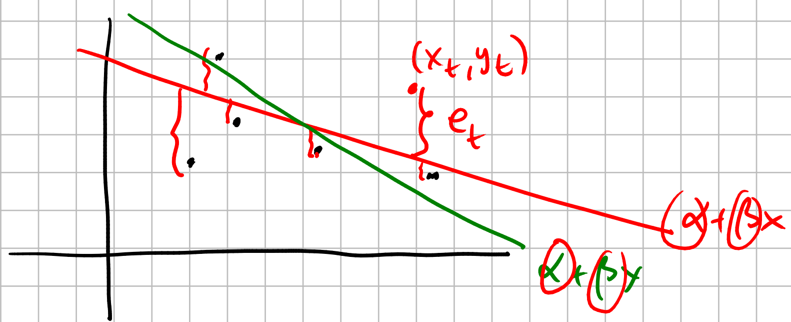
$\{1, 2, 1, 1, 3, 5, 7\}$

mean / average

standard deviation

"the best fitting line"

$\{(1,2), (3,4), (1,-1), (2,-1), (-2,-4), \dots\}$



$$e_t = \underbrace{y_t}_{\text{actual y-value}} - \underbrace{(\alpha + \beta x_t)}_{\text{y-value given by the line}}$$

$$\min_{\alpha, \beta} \frac{1}{T} \sum_{t=1}^{T} e_t^2$$

$$\frac{1}{T}\left(e_1^2 + e_2^2 + \dots + e_T^2\right)$$

t	x_t	y_t	e_t	e_t^2
1				
2				
3				
$\vdots$				
T				

$$\min \frac{1}{T} \sum e_t$$

$$\frac{1}{T} \sum_{t=1}^{T} (y_t - \alpha - \beta x_t)$$

$$\min_{\alpha, \beta} \underline{(y_1 - \alpha - \beta x_1) + (y_2 - \alpha - \beta x_2) + \dots + (y_T - \alpha - \beta x_T)}$$

$$\text{deriv. wrt } \alpha = -1 - 1 - \dots - 1 = -T \overset{\text{FOC}}{=} 0$$

optimal $\alpha, \beta \Rightarrow$

What you have done is called linear regression!

$$\min_{\alpha,\beta} \frac{1}{T}\sum_{t=1}^{T} e_t^2 = \min_{\alpha,\beta} \frac{1}{T}\sum_{t=1}^{T}(y_t-\alpha-\beta x_t)^2$$

$$= \min_{\alpha,\beta} \underbrace{\frac{1}{T}\left[(y_1 \underbrace{-\alpha-\beta x_1}) ^2 + (y_2-\alpha-\beta x_2)^2 + \dots + (y_T-\alpha-\beta x_T)^2\right]}_{L(\alpha,\beta)}$$

$$\frac{\partial L}{\partial \alpha} = \frac{1}{T}\left[2(y_1-\alpha-\beta x_1)(-1) + 2(y_2-\alpha-\beta x_2)(-1) + \dots + 2(y_T-\alpha-\beta x_T)\right]$$

$$\frac{\partial L}{\partial \beta} = \frac{1}{T}\left[2(y_1-\alpha-\beta x_1)(-x_1) + 2(y_2-\alpha-\beta x_2)(-x_2) + \dots + 2(y_T-\alpha-\beta x_T)(-x_T)\right]$$

$$\frac{\partial L}{\partial \alpha} = 0, \quad \frac{\partial L}{\partial \beta} = 0$$

$$-\frac{2}{T}\left[(y_1-\alpha-\beta x_1) + \dots + (y_T-\alpha-\beta x_T)\right] = 0$$

$$-\frac{2}{T}\left[(y_1-\alpha-\beta x_1)x_1 + \dots + (y_T-\alpha-\beta x_T)x_T\right] = 0$$

$$\Rightarrow \begin{cases} (y_1-\alpha-\beta x_1) + \dots + (y_T-\alpha-\beta x_T) = 0 \\ (y_1-\alpha-\beta x_1)x_1 + \dots + (y_T-\alpha-\beta x_T)x_T = 0 \end{cases}$$

$$\Rightarrow \underbrace{y_1+y_2+\dots+y_T}-T\alpha-\beta(x_1+\dots+x_T) = 0$$

$$\boxed{T\mu_y - T\alpha - \beta T\mu_x = 0}$$

$$x_1y_1 + x_2y_2 + \dots + x_Ty_T - \alpha(x_1+x_2+\dots+x_T) - \beta(x_1^2+x_2^2+\dots+x_T^2) = 0$$

$$\boxed{T\sigma_{xy} + \mu_x\mu_y - \alpha T\mu_x - \beta(T\sigma_{xx}+\mu_x^2) = 0}$$

Section 13.4 Exercise 5

(a) $x = 29 - 5p + 4q$ $\qquad$ price A $= p$

$y = 16 + 4p - 6q$ $\qquad$ price B $= q$

costs for firm A $= 5 + x$

costs for firm B $= 3 + 2y$

$\max\limits_{x,y}$ profits $= px + qy - (5+x) - (3+2y)$

will not work

$\downarrow$

express profits in terms of p & q.

$\Pi(\underbrace{p, q}_{\text{prices.}})$

(b) Firm A $\quad \Pi_A(p) = px - (5+x)$

q is taken as given.

$$= p(29 - 5p + 4q) - (5 + 29 - 5p + 4q)$$

$$= 29p - 5p^2 + 4pq - 34 + 5p - 4q$$

$$= 34p - 5p^2 + 4pq - 34 - 4q$$

$\max\limits_{p} \Pi_A(p)$

$$\frac{\partial \Pi_A}{\partial p} = 34 - 10p + 4q = 0$$

$$-10p = -34 - 4q$$

$$p = \frac{17}{5} + \frac{2}{5}q$$

$$\frac{\partial^2 \Pi_A}{\partial p^2} = \dots$$

reaction functions.

$\max\limits_{q} \Pi_B(q)$

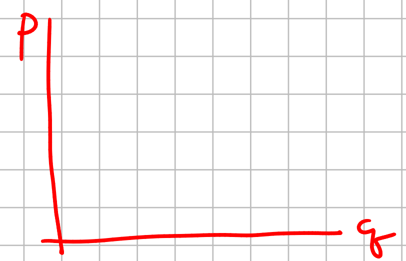
$$q = \_\_\_\_ p.$$

(c) $p = \frac{17}{5} + \frac{2}{5}q$

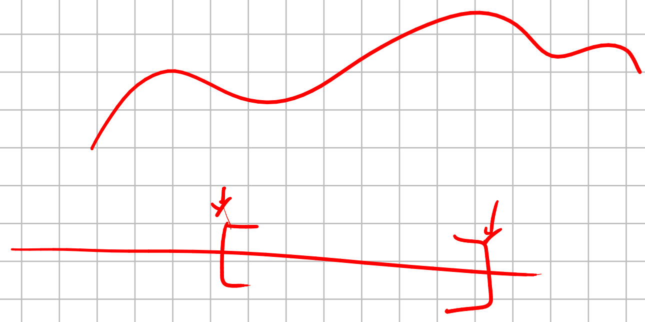
$q = \_\_ + \_\_ p$

$p, q = ?$

(d)



Ch8



Ch17