Functions of one variable (single-variable)
optimization

- what this means in a mathematical sense?
- where to see applications?
- how to actually do it?

Chapter 8 → 13 → 14 → constraints

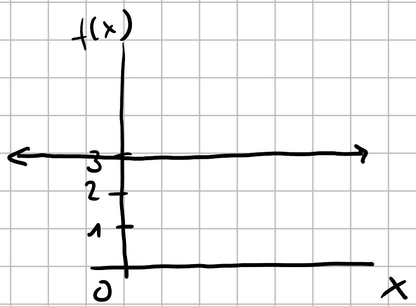
multivariable optimization

Example $f(x) = 3$

Task: Find values of x which will make f the ~~largest~~ smallest.

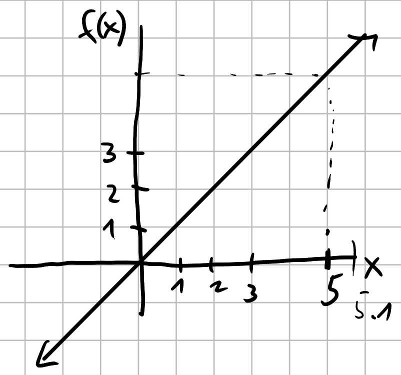
Find values of x which will ~~maximize~~ minimize f.

Any value of $x \in \mathbb{R}$ will ~~maximize~~ minimize f.



$f(x) = x$

Task: Find values of x which will ~~maximize~~ minimize f.



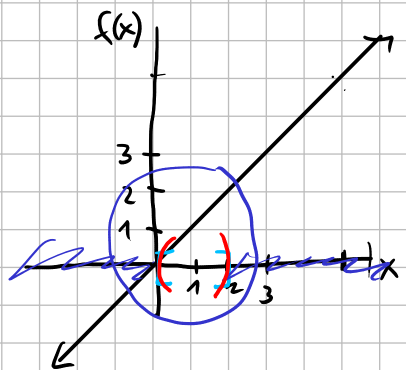
$\pm\infty$ (not numbers!)

$$\lim_{x\to\infty} f(x) = \infty$$

f grows without bound as x increases without bound.

There is no $x \in \mathbb{R}$ which will ~~maximize~~ minimize f.

$f(x) = x$ on $[0,2]$ $(0,2)$ interval to look for maximizers/minimizers



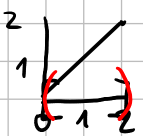
Task: Find values of x in $[0,2]$ $(0,2)$ such that we can maximize f. (minimize)

$x=2$ maximizes f

$x=0$ minimizes f

~~$x=1.99$ maximizes f~~

$x = 1.999$



Claim: $x=1$ maximizes f.

This claim is incorrect. Why?

$x=1.1 \Rightarrow f(1.1) = 1.1 > f(1)$

$\lim_{x\to 2} f(x) = 2$ is correct (here x approaches 2 but is never at 2!)

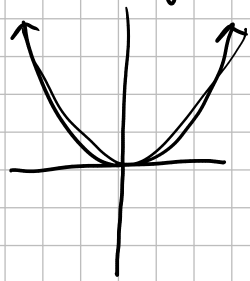
There is no value of $x \in (0,2)$ which will maximize f.

$f(x) = 3-(x-2)^2$

Task: find values of x in $\mathbb{R}$ which will ~~maximize~~ minimize f.

x^2

$-x^2$



$x=2$ maximizes f.

no values of x will minimize f.

$3-((x-2)^2)$

$\Longleftrightarrow$ if and only if

$c \ (\in D)$ is a max pt for f $\Longleftrightarrow$ $f(x) \le f(c)$ for all $x \in D$.

$f(x) = 3-(x-2)^2$

$$(x-2)^2 \ge 0 \quad \text{for all } x \in \mathbb{R} = D$$
$$\Rightarrow \quad -(x-2)^2 \le 0$$
$$\Rightarrow \quad -(x-2)^2+3 \le 0+3$$
$$\Rightarrow \quad f(x) \le \boxed{3} = f(2)$$

inequalities

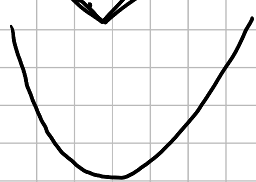
$f(x) \le f(2)$ for all $x \in \mathbb{R}$

By definition of max pt. $x=2$ is a max pt of f.

$g(x) = \sqrt{x-5} - 100$, $x \ge 5$ $\qquad D=[5,+\infty)$

$f(x) = ax^3+bx^2+cx+d$

differentiable function $\qquad$ "no sharp turns", "smooth"



f differentiable
c is max/min
c is interior pt
$\Big\}\ \Longrightarrow\ f'(c)=0.$

$I=[a,b] \qquad (a,b)$

endpoints / boundaries

$f'(c)=0$ is called a necessary condition for a max/min.