$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ treat (a_{11}, a_{12}), (a_{21}, a_{22}) as vectors.

$\downarrow$

$n \times 1$ matrix

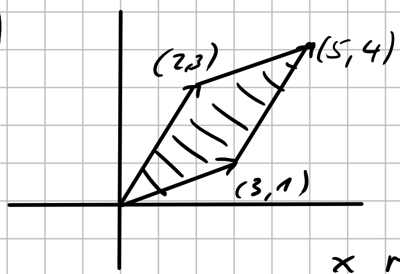
$1 \times n$ matrix

physics

direction

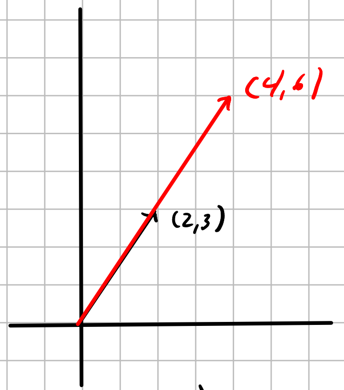
$(2,3)$

$(3,1)$



$(2,3)$

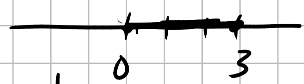
$(4,6)$



$|A|$ rotation?

x real number

$|x|$



$|3| = |3-0|$

$A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$

$|A| = 2(6) - 4(3)$
$= 0$

$A = \begin{pmatrix} 1 & 3 \\ 4 & 8 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} = 4 \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}$

$A = \begin{pmatrix} 100 & 200 \\ 300 & 400 \end{pmatrix} = 100 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$|A| = 100^2 \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} =$ ____

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Determinant of an upper triangular matrix

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{vmatrix} = a_{11} a_{22} a_{33} \dots a_{nn}$$

apply rule.

3x3

$$\begin{vmatrix} a_{11} & \cancel{a_{12}} \overset{0}{} & \cancel{a_{13}} \overset{0}{} \\ a_{21} & a_{22} & \cancel{a_{23}} \overset{0}{} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & \cancel{a_{23}} \overset{0}{} \\ a_{32} & a_{33} \end{vmatrix} \underbrace{- \cancel{a_{12}} \overset{0}{} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + \cancel{a_{13}} \overset{0}{} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}_{=0}$$

section 16.2

$= a_{11}(a_{22} a_{33})$

To calculate determinant, apply elementary row op. to create upper Δar matrices then

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 5 \end{pmatrix}$$

Find $|A|$.

$$\begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & 5 \end{vmatrix} = -\begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 5 \\ 1 & 0 & -1 \end{vmatrix} = -\begin{vmatrix} 2 & 1 & 3 \\ 0 & -3 & -7 \\ 1 & 0 & -1 \end{vmatrix}$$

$R_2 \leftrightarrow R_3$ + Rule D

$R_2 \leftarrow -2R_2 + R_1$ Rule F

$\begin{matrix} 2 & 1 & 3 \\ -2 & -4 & -10 \end{matrix}$

$$= -\begin{vmatrix} 2 & 1 & 3 \\ 0 & -3 & -7 \\ 0 & 1 & 1 \end{vmatrix} = -\begin{vmatrix} 2 & 1 & 3 \\ 0 & -3 & -7 \\ 0 & 0 & -4 \end{vmatrix} = -(2)(-3)(-4) = -24$$

$R_3 \leftarrow -2R_3 + R_1$

$\begin{matrix} 2 & 1 & 3 \\ -2 & 0 & -2 \end{matrix}$

Rule F

$R_3 \leftarrow 3R_3 + R_2$

$\begin{matrix} 0 & -3 & -7 \\ 0 & 3 & 3 \end{matrix}$

application of Rule F is incorrect

differences with respect to Sec 15.6

$$\left(\begin{matrix} 1 & & \\ & 1 & \\ & & 1 \end{matrix} \;\middle|\; \right)$$

no need to produce staircase of 1's!
upper triangular matrix is enough.

---

1 is called identity

$a \cdot 1 = a$ $\qquad a \in \mathbb{R}$

I is identity matrix
square

$$\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$AX = I$

$\underset{2\times3}{A}\ \underset{3\times2}{X} = \underset{2\times2}{I}$ $\qquad$ $\cancel{\underset{3\times2}{X}\ \underset{2\times3}{A} = \underset{3\times3}{I}}$

Example 1 (a) $\quad AX \overset{?}{=} I$ or $XA \overset{?}{=} I$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ has no inverse.

$$\underset{2\times2}{A}\ \underset{2\times2}{X} = \underset{2\times2}{I}$$

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

→ These two matrices would never be equal!

don't match!

$$\underset{A}{\begin{pmatrix} 5 & 6 \\ 5 & 10 \end{pmatrix}} \underset{X}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \underset{I}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 5a+6c & 5b+6d \\ 5a+10c & 5b+10d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 5a+6c = 1 \\ 5b+6d = 0 \\ 5a+10c = 0 \\ 5b+10d = 1 \end{cases}$$

$-4c = 1$
$c = -1/4$

$a =$

$-4d = -1$
$d = 1/4$
$b = ?$

A has an inverse

Because $AX = I \Rightarrow |AX| = |I| \Rightarrow |A||X| = |I|$ Thm 16.4.1

Square matrices.

$\Rightarrow |A||X| = 1$

(*) A has an inverse $\Rightarrow$ $|A| \neq 0$

It also means that

$|A| = 0 \Rightarrow$ A does not have an inverse

A sq. matrix cannot have have more than one inverse (if it exists)

Let X + Y be inverses of A.

$$Y = IY = XAY = XI = X$$

def of identity matrix

A is an inverse of X / X is an inverse of A

$$AX = XA = I$$

since Y is also an inverse of A $(AY = YA = I)$

Therefore, $Y = X$.

A^{-1} the inverse of A (unique notation)

A has inverse

$$AX = I$$
$$A^{-1}AX = A^{-1}I$$
$$IX = A^{-1}$$
$$X = A^{-1}$$

square, inverse exists

$$\boxed{A}x = b$$
$$A^{-1}Ax = A^{-1}b$$
$$x = A^{-1}b$$