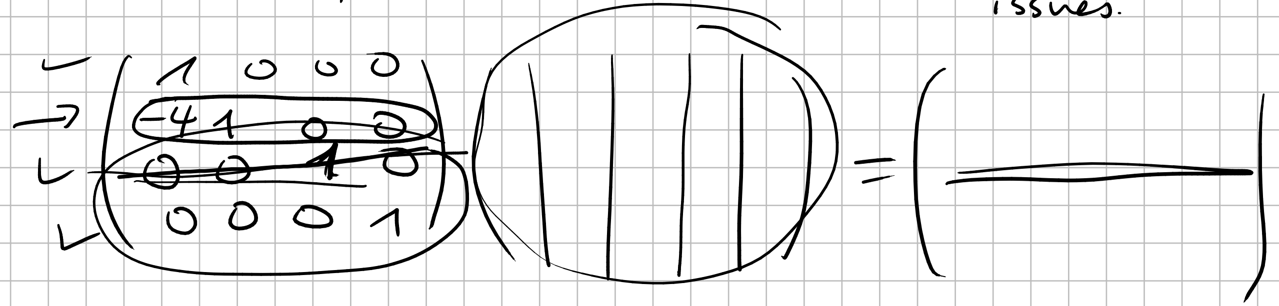
HW01 Exercise 5

Polynomial interpolation

numerical analysis (numerical methods) subject to alot of numerical issues.



Exercise 3

$A = AB$

What you are given : $AB = A$ and $BA = B$

What you need to prove : A is idempotent ($AA = A$).

Goal:

$AA = \dots = A$ $\qquad ABA = (AB)A = A(BA)$

$\boxed{A}A = ABA \overset{(1)}{=} A(BA) = \boxed{AB} = A$ given that $AB = A$

(1) associative law → 15.4

because $AB = A$ (given) $\qquad$ because $BA = B$

$AB = A \Rightarrow \cancel{ABB^{-1} = AB^{-1}}$ not sure that B^{-1} exists

Can we use distributive property? Not really.

$A(B+C) = AB + AC$

$(A+B)C = AC + BC$

$\dfrac{A\cancel{B}}{\cancel{B}} = \dfrac{A}{B}$ does not make sense! Matrices are not necessarily real numbers

matrix addition, multiplication by a scaler,

matrix multiplication

BUT we do NOT have matrix division!

Continuing from last Thursday:

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{2} & -\frac{7}{2} \\ 0 & 0 & \frac{27}{2} & \frac{27}{2} \end{array}\right) \begin{array}{l} \\ \to x_2 - \frac{1}{2} x_3 = -\frac{7}{2} \Rightarrow x_2 = -\frac{7}{2} + \frac{1}{2}(1) = -3 \\ \to \frac{27}{2} x_3 = \frac{27}{2} \Rightarrow x_3 = 1 \end{array}$$

$$\to x_1 + x_2 + 3x_3 = 2$$

$$x_1 + (-3) + 3(1) = 2$$

$$x_1 = 2$$

Example 2

$$\left(\begin{array}{ccc|c} 1 & 3 & -1 & 4 \\ 0 & 1 & -3/5 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \begin{array}{l} \to x_1 + 3x_2 - x_3 = 4 \\ \to x_2 - \frac{3}{5} x_3 = \frac{1}{5} \\ \to 0 = 0 \\ \to 0 = 0 \end{array}$$

always true

x_3 is a "free variable"

$$x_3 = t \in \mathbb{R}$$

$$x_2 = \frac{3}{5}t + \frac{1}{5}$$

$$x_1 = 4 + t - 3\left(\frac{3}{5}t + \frac{1}{5}\right) = -\frac{4}{5}t + \frac{17}{5}$$

solution set is not unique

$$(x_1, x_2, x_3) = \left(-\frac{4}{5}t + \frac{17}{5}, \frac{3}{5}t + \frac{1}{5}, t\right)$$

$$= \left(-\frac{4}{5}, \frac{3}{5}, 1\right) t + \left(\frac{17}{5}, \frac{1}{5}, 0\right)$$

When does a system of linear equations have a solution?
↳ one of the main questions of linear algebra

$$\boxed{A}x = b$$

Section 15.6
Example 3

$$0 = 2a - 3b + c$$

~~$a = 0$~~
~~$b = 0$~~
$c = 1$

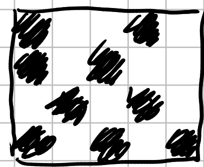
not all (a, b, c) will guarantee

$a, b \in \mathbb{R}$ $\quad (a+b)(a+b) = a^2 + 2ab + b^2$

A, B are matrices (of the appropriate type)

$(A+B)(A+B) = AA + AB + BA + BB$

Sparsity / Sparse matrices lots of zeros in a matrix



$(x+5)(x+2) = 0 \Rightarrow x = -5 \lor x = -2$

NOT apply to matrices!

Section 15.4 Exercise 2

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{A}\; x$$

"linear"

$$(x \;\; y)\underbrace{\begin{pmatrix} a & d \\ d & b \end{pmatrix}}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= (ax + dy \quad dx + by)\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= (ax+dy)x + (dx+by)y$$

$$= ax^2 + dxy + dxy + by^2$$

$$= ax^2 + 2dxy + by^2$$

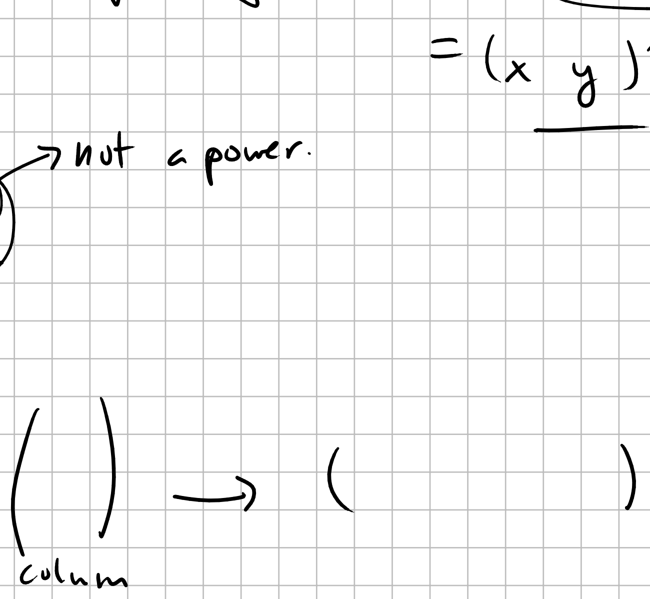
$$x^2 - 2xy + y^2$$

$$= (x \;\; y)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

A' or A^T → not a power.

↑ ambiguous.

Transpose



$$\underset{(n\times 1)}{\mathbf{x}} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \text{or} \quad (x_1, x_2, \ldots, x_n)'$$

squared length according to some notion of distance

$$\underset{1\times n}{\mathbf{x}'}\ \underset{n\times 1}{\mathbf{x}} = \underset{1\times 1}{\_\_\_} \quad \text{scalar}$$

$$\underset{n\times 1}{\mathbf{x}}\ \underset{1\times n}{\mathbf{x}'} = \underset{n\times n}{\_\_\_} \quad \text{matrix}$$

$$\mathbf{x}'\mathbf{x} = (x_1 \ x_2 \ \ldots \ x_n)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$= x_1^2 + x_2^2 + \cdots + x_n^2$$

$$= \left[\sqrt{(x_1-0)^2 + (x_2-0)^2 + \cdots + (x_n-0)^2}\right]^2$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Section 16.1

(2)

$$x_1 = \frac{\quad}{a_{11}a_{22} - a_{12}a_{21}}$$

$$x_2 = \frac{\quad}{a_{11}a_{22} - a_{12}a_{21}}$$

x_1 & x_2 might not even exist!

When? $a_{11}a_{22} - a_{12}a_{21} = 0$

$a_{11}a_{22} - a_{12}a_{21} \approx 0$

0.00000001

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{?}{-1} \\ \frac{?}{-1} & 1 \end{pmatrix} \checkmark \qquad 1(1) - \frac{(-1)(-1)}{1} = 0$$

$$2\begin{pmatrix} 1 & \frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix} \checkmark \qquad a_{11}a_{22} - a_{12}a_{21} = 0$$