

AB is not always defined

$$AB \neq BA$$

main differences between matrix multiplication & ordinary multiplication

$$2 \cdot 5 = 5 \cdot 2$$

Matrix multiplication is NOT commutative.

Section 15.3 Example 3 → dynamical systems

What would be the new market shares after a year?

	left firm A	taken firm B	taken firm C	
new market share A	$0.85(0.2)$	$+ 0.1(0.6)$	$+ 0.1(0.2)$	$= 0.25$
B	$0.05(0.2)$	$+ 0.55(0.6)$	$+ 0.05(0.2)$	$= 0.35$
C	$0.10(0.2)$	$+ 0.35(0.6)$	$+ 0.85(0.2)$	$= 0.40$

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
 a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
 a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
 \end{aligned}$$

$$\begin{pmatrix} 0.85 & 0.10 & 0.10 \\ 0.05 & 0.55 & 0.05 \\ 0.10 & 0.35 & 0.85 \end{pmatrix}
 \begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.35 \\ 0.40 \end{pmatrix}$$

multiplying 2 matrices

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{\text{coeff matrix } A}
 \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\text{unknowns } x} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}_{\text{constants } b}$$

market dynamics → Will things stabilize in the long run?

T_S mkt share after 1 yr.

$T(T_S)$ mkt share after 2 yrs.

\vdots

$\lim_{n \rightarrow \infty} T^n_S = \text{finite?}$

$T(T_S) = TT_S = T^2_S$
 \downarrow
 T matrix

T transition matrix (model dynamics)

economics T (movements from employed to unemployed)

Systems of linear equations \rightarrow matrix form

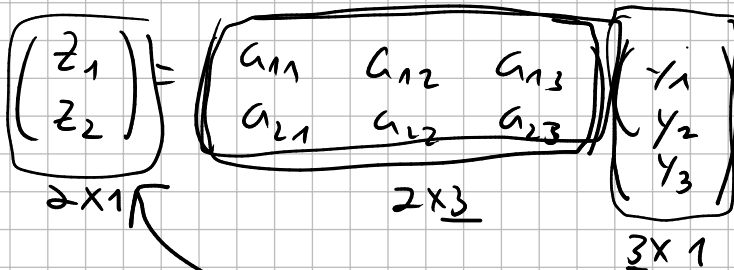
$Ax = b$
 $(n \times n) (n \times 1) \downarrow (n \times 1)$

* Matrix multiplication is "roughly" a composition of linear transformations (linear functions)

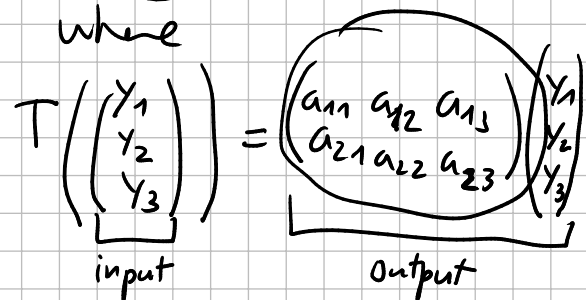
* Matrices are "roughly" linear functions.

$Z_1 = a_{11}Y_1 + a_{12}Y_2 + a_{13}Y_3$

$Z_2 = a_{21}Y_1 + a_{22}Y_2 + a_{23}Y_3$



Let T be a function where



$Y_1 = b_{11}X_1 + b_{12}X_2$
 $Y_2 = b_{21}X_1 + b_{22}X_2$
 $Y_3 = b_{31}X_1 + b_{32}X_2$

$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

$$S \left(\underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\text{Input}} \right) = \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_{\text{Output}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$S \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right) = \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_{\mathbf{B}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\mathbf{x}}$$

$$S(\mathbf{x}) = \mathbf{B}\mathbf{x} \quad \text{Output } (3 \times 1)$$

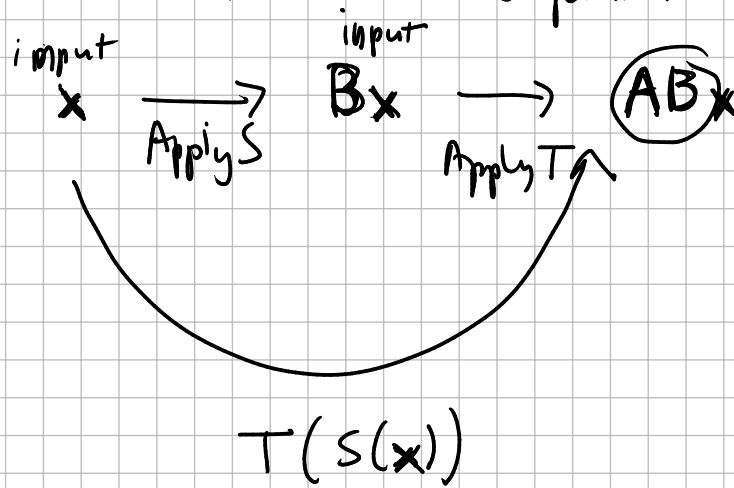
$$T(\mathbf{y}) = \mathbf{A}\mathbf{y} \quad (2 \times 1)$$

$$T \left(\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \right) = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}}_{\mathbf{y}}$$

$$T(S(\mathbf{x})) = \boxed{\mathbf{AB}}\mathbf{x}$$

↑ composition

↓ matrix multiplication!



Section 16.9 Example 1

α alpha

β beta

γ gamma

Section 12.11 Example 1

$$\begin{aligned} 5u + 5v &= 2x - 3y \\ 2u + 4v &= 3x - 2y \end{aligned}$$

Systems of 2 lin eqns in functions.

We are not necessarily interested in the solution itself

but we are interested in how the solution is affected by a change in x (for example).

$$y = f(x) \quad \text{approx chge in } x$$

$$dy = f'(x) dx \quad \text{approx chge in } y \rightarrow \text{percent chge in } x$$

$$5 du + 5 dv = 2 dx - 3 dy$$

$$2 du + 4 dv = 3 dx - 2 dy$$

Assume that only x changes ^(dx ≠ 0) but y does not. ^(dy = 0) How will u and v be affected?

$$5 du + 5 dv = 2 dx$$

$$2 du + 4 dv = 3 dx$$

$$5 \frac{du}{dx} + 5 \frac{dv}{dx} = 2$$

$$2 \frac{du}{dx} + 4 \frac{dv}{dx} = 3$$

$$\begin{matrix} A & * & b \\ \left(\begin{array}{cc} 5 & 5 \\ 2 & 4 \end{array} \right) & \left(\begin{array}{c} \frac{du}{dx} \\ \frac{dv}{dx} \end{array} \right) & = \left(\begin{array}{c} 2 \\ 3 \end{array} \right) \end{matrix}$$

* Calculus + linear algebra!

A lot of time was spent setting up / recognizing situations for which we have systems of linear equations!

How to solve? Section 15.6

Systematic way → Gaussian elimination

Example 1

$$2x_2 - x_3 = -7$$

$$x_1 + 2x_2 + 3x_3 = 2$$

$$-3x_1 + 2x_2 + 2x_3 = -10$$

$$\Rightarrow A = \begin{pmatrix} 0 & 2 & -1 \\ 1 & 2 & 3 \\ -3 & 2 & 2 \end{pmatrix} \quad b = \begin{pmatrix} -7 \\ 2 \\ -10 \end{pmatrix}$$

$$A \quad | \quad b \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

augmented matrix

$$\left(\begin{array}{ccc|c} 0 & 2 & -1 & -7 \\ 1 & 2 & 3 & 2 \\ -3 & 2 & 2 & -10 \end{array} \right)$$

goal is to hopefully reach the following pattern

$$\left(\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{array} \right)$$

*'s could be anything.

NOTE but this pattern is not always going to happen.

$$\left(\begin{array}{ccc|c} 0 & 2 & -1 & -7 \\ 1 & 1 & 3 & 2 \\ -3 & 2 & 2 & -10 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 2 & -1 & -7 \\ -3 & 2 & 2 & -10 \end{array} \right)$$

exchange row 1 with row 2

new row 2 is half of the old row 2

$$R_2 \leftarrow \frac{1}{2} R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -1/2 & -7/2 \\ -3 & 2 & 2 & -10 \end{array} \right)$$

$$\begin{array}{r} 3 \\ -3 \\ \hline 3 \quad 2 \quad 1 \quad -7 \\ \quad 5 \quad 11 \quad -4 \end{array}$$

$$R_3 \leftarrow 3R_1 + R_3 \quad \text{or} \quad R_1 + \frac{1}{3}R_3 \quad \text{or} \quad R_3 + 3R_1$$

new row 3 is obtained by multiplying row 1 by 3 and adding to row 3

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -1/2 & -7/2 \\ 0 & 5 & 11 & -4 \end{array} \right)$$

$$R_3 \leftarrow -5R_2 + R_3 \quad \text{or} \quad \frac{1}{5}R_3 - R_2$$

$$\begin{array}{ccc|c} 0 & -5 & 5 & 35/2 \\ 0 & 5 & 11 & -4 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & -1/2 & -7/2 \\ 0 & 0 & 27/2 & 27/2 \end{array} \right)$$

Group HW tomorrow at Nexities
groupings will be available at amspace (tomorrow)