Section 15.1 Example 1.

- setting up a system of linear equations
- verifying whether a solution satisfies a particular system of linear equations (rat poison principle)

$(2, -1, 0) \to$ is this a solution to

$x_1 \quad x_2 \quad x_3$

$$\begin{cases} 2x_1 + 3x_2 + 4x_3 = 1 \\ 3x_1 + 4x_2 + 5x_3 = 2 \\ 4x_1 + 5x_2 + 6x_3 = 3 \end{cases} ?$$

You are not required to actually solve for x_1, x_2, x_3!

ordering matters $(2, 0, -1) \leftarrow$ Is this a solution?

$2(2) + 3(-1) + 4(0) \overset{?}{=} 1$

$4 - 3 \overset{?}{=} 1$

$1 \overset{\checkmark}{=} 1$

$3(2) + 4(-1) + 5(0) \overset{?}{=} 2$

$6 - 4 \overset{?}{=} 2$

$2 \overset{\checkmark}{=} 2$

$4(2) + 5(-1) + 6(0) \overset{?}{=} 3$

$8 - 5 \overset{?}{=} 3$

$3 \overset{\checkmark}{=} 3$

Therefore $(2, -1, 0)$ is a solution to

Remark. The general solution is $(x_1, x_2, x_3) = (2-t, -1-2t, t)$ where t is any real number.

If $t = 0$, then $(2-0, -1-2(0), 0) = (2, -1, 0)$ is a particular solution.

Is $(1, -1, \frac{1}{2})$ a solution to the linear system above?

$2(1) + 3(-1) + 4(\frac{1}{2}) \overset{?}{=} 1$

$2 - 3 + 2 \overset{?}{=} 1$

$1 \overset{\checkmark}{=} 1$

$3(1) + 4(-1) + 5(\frac{1}{2}) \overset{?}{=} 2$

$3 \underbrace{- 4 + \frac{5}{2}}_{1.5} \neq 2$

No. $(1, -1, \frac{1}{2})$ is not a solution to the linear system above.

Section 15.1 ~~Exercise 6~~

$c = 0.712y + 95.05$

$S = 0.158(c+x) - 34.30$

$y = c + x - S$

$x = 93.53$

convert this into the form (1)

x, y, S, c

$\underline{0}x + \underline{-0.712}y + \underline{0}s + \underline{1}c = \underline{95.05}$

$\underline{-0.158}x + \underline{0}y + \underline{1}s + \underline{-0.158}c = \underline{-34.3}$

$S = 0.158(c+x) - 34.3$

$\boxed{S = 0.158c + 0.158x - 34.3}$

$\underline{-1}x + \underline{1}y + \underline{1}s + \underline{-1}c = \underline{0}$

$y = c + 1x - S$

$\underline{1}x + \underline{0}y + \underline{0}s + \underline{0}c = \underline{93.53}$

$x = 93.53$

$$\begin{array}{rrrrcl}
 & -0.712y & & +c & = & 95.05 \\
-0.158x & & +S & -0.158c & = & -34.3 \\
-x & +y & +S & -c & = & 0 \\
x & & & & = & 93.53
\end{array}$$

*

$x = 93.53$

Reduce the system above to

$$\begin{array}{rrrcl}
-0.712y & & +c & = & 95.05 \\
 & S & -0.158c & = & -34.3 - 0.158(93.53) \\
1y & +S & -c & = & 93.53
\end{array}$$

$$
\left\{
\begin{array}{l}
-0.712y + c = 95.05 \\
s - 0.158c = -34.3 - 0.158(93.53) \\
* \quad 1y + s - c = 93.53
\end{array}
\right.
$$

Multiply 3rd row by $+0.712$

$$+0.712y + 0.712s - 0.712c = 93.53(0.712)$$

Add 1st and ↑ row

$$+0.712s - 0.288c = 95.05 + 93.53(0.712)$$

Reduce the system above to

$$
\left\{
\begin{array}{l}
-0.712y + c = 95.05 \\
\underline{s} - 0.158c = -34.3 - 0.158(93.53) \\
+0.712\underline{s} - 0.288c = 161.6434
\end{array}
\right.
$$

Next steps: Multiply 2nd row by -0.712 then add to 3rd row to eliminate s

$\Rightarrow$ result will be a way to find c.

Section 15.6

$$
\begin{array}{l}
x_1 + bx_2 + cx_3 = d \\
\boxed{x_2} + ex_3 = f \\
x_3 = g
\end{array}
$$

Square matrices → main diagonal

$$
\begin{pmatrix}
a_{11} & & & \\
& a_{22} & & \\
& & \ddots & \\
& & & a_{nn}
\end{pmatrix}
$$

a_{ii} (ith row, ith column entry of A)

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

coef matrix $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$ $\quad \mathbf{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$\mathbf{A}\mathbf{x} = \mathbf{b}$

Two matrices $A + B$ are equal when

- they both have the same number of columns & rows
- $a_{ij} = b_{ij}$ for all i, j

(i,j)th entry of A should match (i,j) entry of B for all i, j

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1+1 & 3+0 \\ 2+0 & 4+2 \end{pmatrix}$$

A 2×2 $\quad$ B 2×2

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \text{is undefined}$$

2×2 $\quad$ 3×3

$\alpha \mathbf{A}$ (multiplying a matrix $\mathbf{A}$ by a scalar α)

α → constant / real number

A scalar is a 1×1 matrix

$$2\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

Matrix Multiplication (def)

AB (A first, then B)

$m \times n$ $\quad$ $n \times p$

are the same

matrix mult. works if the number of cols in the first matrix matches the number of rows in the 2nd matrix

$$c_{ij} = \sum_{r=1}^{n} \boxed{a_{ir} b_{rj}}$$

ith row, rth column entry of A

rth row, jth column of B

$\sum_{r=1}^{n}$ (summation notation)

signal for you to add things

(i,j)th entry of $C \Rightarrow$ add up things $\Rightarrow$ which things? $a_{ir} b_{rj}$

$r=1$	$a_{i1} b_{1j}$
$r=2$	$a_{i2} b_{2j}$
$r=3$	$a_{i3} b_{3j}$
$\vdots$	
$r=n$	$a_{in} b_{nj}$

TOTAL $\rightarrow c_{ij}$