

Section 15.1 Example 1.

- setting up a system of linear equations
- verifying whether a solution satisfies a particular system of linear equations (rat poison principle)

$$\begin{array}{c} \boxed{(2, -1, 0)} \rightarrow \text{is this a solution to} \\ \uparrow \quad \uparrow \quad \uparrow \\ x_1 \quad x_2 \quad x_3 \end{array} \quad \begin{cases} 2x_1 + 3x_2 + 4x_3 = 1 \\ 3x_1 + 4x_2 + 5x_3 = 2 \\ 4x_1 + 5x_2 + 6x_3 = 3 \end{cases} ?$$

You are not required to actually solve for x_1, x_2, x_3 !

ordering matters $\underline{(2, 0, -1)} \leftarrow$ (is this a solution?)

$$\begin{aligned} 2(2) + 3(-1) + 4(0) &\stackrel{?}{=} 1 \\ 4 - 3 &\stackrel{?}{=} 1 \\ 1 &\leq 1 \end{aligned} ?$$

$$\begin{aligned} 4(2) + 5(-1) + 6(0) &\stackrel{?}{=} 3 \\ 8 - 5 &\stackrel{?}{=} 3 \\ 3 &\leq 3 \end{aligned}$$

$$\begin{aligned} 3(2) + 4(-1) + 5(0) &\stackrel{?}{=} 2 \\ 6 - 4 &\stackrel{?}{=} 2 \\ 2 &\leq 2 \end{aligned}$$

Therefore $\underline{(2, -1, 0)}$ is a solution to

Remark. The general solution is $(x_1, x_2, x_3) = (2-t, -1-2t, t)$ where t is any real number.

If $t=0$, then $(2-0, -1-2(0), 0) = (2, -1, 0)$ is a particular solution.

Is $(1, -1, \frac{1}{2})$ a solution to the linear system above?

$$\begin{aligned} 2(1) + 3(-1) + 4\left(\frac{1}{2}\right) &\stackrel{?}{=} 1 \\ 2 - 3 + 2 &\stackrel{?}{=} 1 \\ 1 &\leq 1 \end{aligned}$$

$$\begin{aligned} 3(1) + 4(-1) + 5\left(\frac{1}{2}\right) &\stackrel{?}{=} 2 \\ 3 - 4 + \frac{5}{2} &\neq 2 \\ \underbrace{\hspace{2cm}}_{7.5} \end{aligned}$$

No. $(1, -1, \frac{1}{2})$ is not a solution to the linear system above.

Section 15.1 Exercise 6

$$\begin{aligned} c &= 0.712y + 95.05 \\ s &= 0.158(c+x) - 34.30 \\ y &= c+x-s \\ x &= 93.53 \end{aligned}$$

convert this into the form (1)
x, y, s, c

$$0x + \underline{-0.712}y + \underline{0}s + \underline{1}c = \underline{95.05}$$

$$\underline{-0.158}x + \underline{0}y + \underline{1}s + \underline{-0.158}c = \underline{-34.3}$$

$$s = 0.158(c+x) - 34.3$$

$$s = 0.158c + 0.158x - 34.3$$

$$\underline{-1}x + \underline{1}y + \underline{1}s + \underline{-1}c = \underline{0}$$

$$y = c+x-s$$

$$\underline{1}x + \underline{0}y + \underline{0}s + \underline{0}c = \underline{93.53}$$

$$x = 93.53$$

	$-0.712y$	$+c$	$= 95.05$
$-$	$-0.158x$	$+s$	$-0.158c = -34.3$
$-$	$-x + y$	$+s$	$-c = 0$
$*$	x		$= 93.53$

$$x = 93.53$$

Reduce the system above to

	$-0.712y$	$+c$	$= 95.05$
		$s - 0.158c$	$= -34.3 - 0.158(93.53)$
	$1y$	$+s - c$	$= 93.53$

$$\begin{array}{r}
 -0.712y \quad + C = 95.05 \\
 S - 0.158C = -34.3 - 0.158(93.53) \\
 * \quad 1y + S - C = 93.53
 \end{array}$$

Multiply 3rd row by $+0.712$

$$+0.712y + 0.712S - 0.712C = 93.53(0.712)$$

Add 1st and \uparrow row

$$+0.712S - 0.288C = 95.05 + 93.53(0.712)$$

Reduce the system above to

$$\begin{array}{r}
 -0.712y \quad + C = 95.05 \\
 \underline{S} - 0.158C = -34.3 - 0.158(93.53) \\
 +0.712\underline{S} - 0.288C = 161.6434
 \end{array}$$

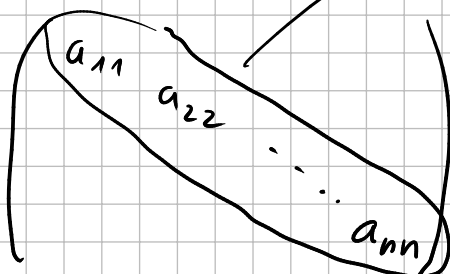
Next steps: Multiply 2nd row by -0.712 then add to 3rd row to eliminate S

\Rightarrow result will be a way to find C .

Section 15.6

$$\begin{array}{l}
 x_1 + 5x_2 + Cx_3 = d \\
 \boxed{x_2} + ex_3 = f \\
 \textcircled{x_3 = g}
 \end{array}$$

Square matrices \rightarrow main diagonal



a_{ij} (ith row, ith column entry of A)

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned}$$

coef matrix $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$ $b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$$Ax = b$$

Two matrices A & B are equal when

- they both have the same number of columns & rows
- $a_{ij} = b_{ij}$ for all i, j

(i, j) th entry of A should match (i, j) entry of B for all i, j

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1+1 & 3+0 \\ 2+0 & 4+2 \end{pmatrix}$$

A B
 2×2 2×2

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \text{is undefined}$$

2×2 3×3

αA (multiplying a matrix A by a scalar α)

↓
constant / real number

A scalar is a 1×1 matrix

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

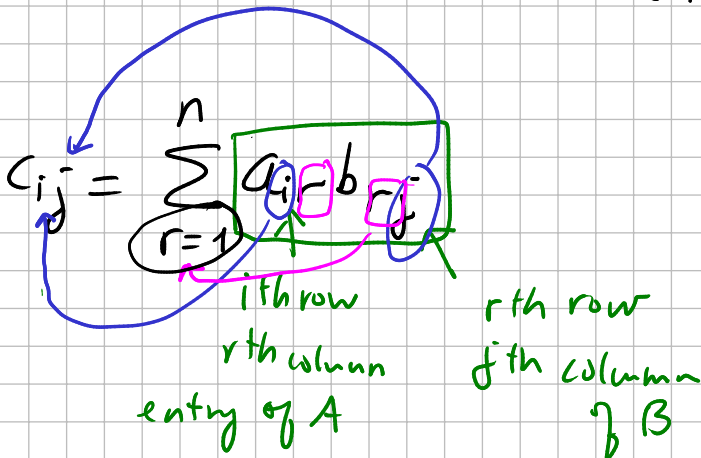
Matrix Multiplication (def)

AB (A first, then B)

$\uparrow \quad \uparrow$
 $m \times n \quad n \times p$
 $\uparrow \quad \uparrow$
 are the same

matrix mult. works if

the number of cols in the first matrix matches the number of rows in the 2nd matrix



$$\sum_{r=1}^n$$

(Summation notation)

Signal for you to add things

(i,j)th entry of C \Rightarrow add up things \Rightarrow which things?

