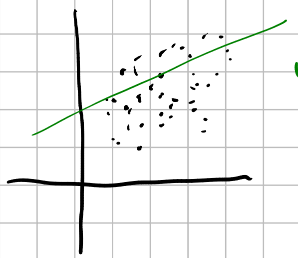
Linear algebra $\rightarrow$ study of systems of linear equations.

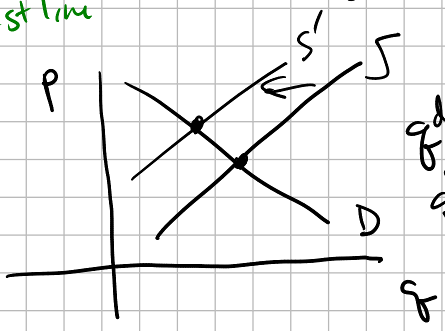
- what it is
- where you might have seen it
- how to do it

$\hookrightarrow$ where else you could see it?

$$\begin{cases} x+y=1 \\ x-y=2 \end{cases}$$

The sum of two unknown numbers is 1.
The difference of two unknown numbers is 2.
What are those two unknown numbers?





$$q^d = 50 - 2p \rightarrow q^d = \alpha_1 + \beta_1 p$$
$$q^s = \boxed{10+3}p \qquad q^s = \alpha_2 + \beta_2 p$$

Comparative statics

machine learning (MML Book)

---

m = the number of equations
n = the number of unknowns

$m > n$, $m = n$, $m < n$ all three cases are covered.

$\underbrace{x_1, x_2, \ldots, x_n}_{n \text{ of them}} \leftarrow$ unknowns

$x_1 \rightarrow$ the first unknown
subscript $\rightarrow$ label
indices

coefficients

$$\boxed{a_{11}}\, x_1 + \boxed{a_{12}}\, x_2 + \cdots + \boxed{a_{1n}}\, x_n = b_1$$

unknowns

two indices
a_{11} not a sub eleven!
row column

$a_{11}, \ldots, a_{mn}, b_1, \ldots, b_m$ are all constants (typically known)

Example x_1, x_2 unknowns

$$x_1 + x_2 = 1$$
$$0x_1 + x_2 = 3$$

$m = 2$ $\quad n = 2$

$a_{11} = 1 \quad b_1 = 1$
$a_{12} = 1 \quad b_2 = 3$
$a_{21} = 0$
$a_{22} = 1$

$\Rightarrow x_1 = 1-3 = -2$

A solution to $\begin{aligned} x_1 + x_2 &= 1 \\ x_2 &= 3 \end{aligned}$ is $(x_1, x_2) = (-2, 3)$

$s_1 \quad s_2$

Example

$x_1 + x_2 = 1$

unknowns ?
m ?
n ?
⋮

$m = 1$
$n = 2$

$a_{11} = 1 \qquad b_1 = 1$
$a_{12} = 1$

$$\underset{1}{\overset{\shortparallel}{a_{11}}} x_1 + \underset{1}{\overset{\shortparallel}{a_{12}}} x_2 = \underset{1}{\overset{\shortparallel}{b_1}}$$

This is an example of a system of linear equations with more than one solution.

$(x_1, x_2) = (-1, 2)$
$(x_1, x_2) = (1, 0)$
$(x_1, x_2) = (1-t, t)$ $\quad t \in \mathbb{R}$ (one degree of freedom)

choose t freely

Example $\quad m=2, n=2$ $\qquad$ m, n, a's ? $\quad$ b's ?

$$\left.\begin{aligned} x_1 + 2x_2 &= 1 \\ \frac{2x_1 + 4x_2}{2} &= \frac{0}{2} \\ x_1 + 2x_2 &= 0 \end{aligned}\right\} \Rightarrow 1 = 0 \text{ which is not true.}$$

This is an example of an inconsistent system of linear equations.

Rat poison principle (Elizabeth Meckes)